

Power counting in relativistic baryon chiral perturbation theory

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We consider a renormalization scheme for relativistic baryon chiral perturbation theory which provides a simple and consistent power counting for renormalized diagrams. As an application we discuss the chiral expansion of the nucleon mass.

The effective field theory of (pseudo) Goldstone bosons [1,2] was extended by Gasser, Sainio, and Švarc to include also processes involving one external nucleon [3] (for a recent review see, e.g., [4]). One of the findings in their scheme was that higher-loop diagrams can contribute to terms as low as $\mathcal{O}(q^2)$, where q generically denotes a small expansion parameter such as, e.g., the pion mass. This problem has been solved in the heavy-baryon formulation of ChPT [5]. Although this approach leads to a straightforward power counting, its disadvantage is that, in some cases, it does not provide the correct analytic behavior even in the threshold regime [6]. Several methods have been suggested to reconcile power counting with the constraints of analyticity in the relativistic approach [7,8,9,10,11,12].

The most general effective Lagrangian includes all possible interaction terms which are compatible with the underlying symmetries and thus provides us with all the required counterterms. Since the finite parts of the counterterms are arbitrary, one has the freedom of choosing a suitable renormalization condition. In this work we choose the finite parts of the counterterms so that their contributions precisely cancel those parts of the loop diagrams which violate the power counting. This leads us to a simple and consistent power counting for the renormalized diagrams of a relativistic approach [12]. As an example we consider the nucleon self energy.

We use the standard power counting of Ref. [13] together with the Lagrangian of Ref. [14].

At $\mathcal{O}(q^4)$, the self energy receives contact contributions Σ_{contact} from $\mathcal{L}_{\pi N}^{(2)}$ and $\mathcal{L}_{\pi N}^{(4)}$ as well as the one-loop contributions of Fig. 1,

$$\Sigma = \Sigma_{\text{contact}} + \Sigma_a + \Sigma_b + \Sigma_c, \quad \Sigma_{\text{contact}} = -4M^2 c_1^0 - 2M^4 (8e_{38}^0 + e_{115}^0 + e_{116}^0), \quad (1)$$

$$\Sigma_a = -\frac{3g_{A0}^{\circ 2}}{4F_0^2} \left\{ (\not{p} + m) I_N + M^2 (\not{p} + m) I_{N\pi}(-p, 0) \right\}$$

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$$-\frac{(p^2 - m^2)\not{p}}{2p^2} [(p^2 - m^2 + M^2)I_{N\pi}(-p, 0) + I_N - I_\pi] \Big\}, \quad (2)$$

$$\Sigma_b = -4M^2 c_1^0 \frac{\partial \Sigma_a}{\partial m}, \quad \Sigma_c = 3 \frac{M^2}{F_0^2} \left(2c_1^0 - c_3^0 - \frac{p^2}{m^2} \frac{c_2^0}{n} \right) I_\pi, \quad (3)$$

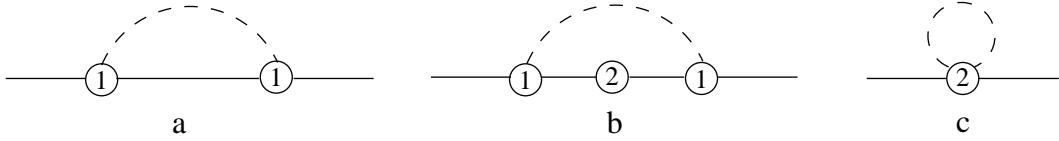


Figure 1. One-loop contributions to the nucleon self-energy at $\mathcal{O}(q^4)$.

where the expressions for the loop integrals read

$$I_{N\pi}(p, 0) = 2\bar{\lambda} - \frac{1}{16\pi^2} + \frac{\sqrt{4m^2M^2 - (p^2 - m^2 - M^2)^2}}{16\pi^2 p^2} \arccos \left(\frac{m^2 + M^2 - p^2}{2mM} \right) + \frac{p^2 - m^2 + M^2}{16\pi^2 p^2} \ln \left(\frac{M}{m} \right), \quad (4)$$

$$I_\pi = 2M^2 \bar{\lambda} + \frac{M^2}{8\pi^2} \ln \left(\frac{M}{m} \right), \quad I_N = 2m^2 \bar{\lambda}, \quad \bar{\lambda} = \frac{m^{n-4}}{(4\pi)^2} \left\{ \frac{1}{n-4} - \frac{1}{2} [\ln(4\pi) + \Gamma'(1) + 1] \right\}. \quad (5)$$

The renormalization of the loop diagrams is performed in two steps. First we render the diagrams finite by applying the subtraction scheme used by Gasser and Leutwyler [2,3] which we denote by modified minimal subtraction scheme of ChPT ($\overline{\text{MS}}$) amounting to dropping the terms proportional to $\bar{\lambda}$ in the loop integrals. We choose the renormalization parameter $\mu = m$. In a second step, a given $\overline{\text{MS}}$ -renormalized diagram is written as the sum of a subtracted diagram which satisfies the power counting and a remainder which violates the power counting and thus needs to be subtracted. We expand the couplings of the $\overline{\text{MS}}$ scheme in terms of the couplings of our extended on-mass-shell (EOMS) scheme, thus generating finite counterterms responsible for additional finite subtractions. These counterterms are fixed so that the net result of combining the counterterm diagrams with those parts of the $\overline{\text{MS}}$ -renormalized diagrams which violate the power counting are of the same order as the subtracted diagram. Hence the sum of an $\overline{\text{MS}}$ -renormalized diagram and the corresponding counterterm diagram satisfies the power counting.

For the case at hand, we determine the subtraction terms by first expanding the integrands and coefficients in Eqs. (2) and (3) in powers of M^2 , $\not{p} - m$, and $p^2 - m^2$. In this expansion we keep all the terms having a chiral order which is smaller than what is suggested by the power counting for the given diagram. We then obtain

$$\Sigma_{r,a+b+c}^{\text{subtr}} = \frac{3g_{A_r}^{\circ 2}}{32\pi^2 F_r^2} \left[mM^2 - \frac{(p^2 - m^2)^2}{4m} \right] + \frac{3c_1^r g_{A_r}^{\circ 2} M^2}{8\pi^2 F_r^2} \left[m(\not{p} + m) - \frac{3}{2}(p^2 - m^2) \right]. \quad (6)$$

We fix the corresponding counterterms so that they exactly cancel the expression given by Eq. (6). Finally, the renormalized self-energy expression is obtained by subtracting Eq. (6) from the $\widetilde{\text{MS}}$ -subtracted versions of Eqs. (2) and (3) and replacing the $\widetilde{\text{MS}}$ -renormalized couplings with the ones of our EOMS scheme.

The physical nucleon mass at order $\mathcal{O}(q^4)$ can be written as [15,16]

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln \left(\frac{M}{m} \right) + k_4 M^4 + \mathcal{O}(M^5). \quad (7)$$

In the EOMS renormalization scheme the coefficients k_i are given by

$$\begin{aligned} k_1 &= -4c_1, \quad k_2 = -\frac{3g_A^{\circ 2}}{32\pi F^2}, \quad k_3 = \frac{3}{32\pi^2 F^2} \left(8c_1 - c_2 - 4c_3 - \frac{g_A^{\circ 2}}{m} \right), \\ k_4 &= \frac{3g_A^{\circ 2}}{32\pi^2 F^2 m} (1 + 4c_1 m) + \frac{3}{128\pi^2 F^2} c_2 - 16e_{38} - 2e_{115} - 2e_{116}. \end{aligned} \quad (8)$$

Comparing with Ref. [8], we see that the lowest-order correction and those terms which are nonanalytic in the quark mass \hat{m} coincide, but the analytic k_4 term is different. This is due to the usage of a different renormalization scheme and hence the difference between the two results is compensated by different values of the renormalized parameters.

It is straightforward to use our approach in an iterative procedure to renormalize higher-order loop diagrams. Finally, our renormalization scheme is neither restricted to the single-nucleon sector nor to the interaction of Goldstone bosons with fermions [17].

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